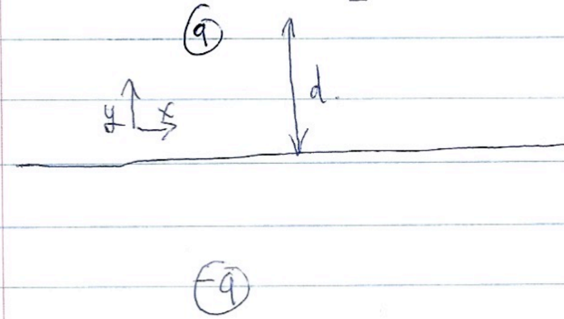
Jackson

2.1 (a) $V(x,y) = kq\left[\left(x^2+(d-y)^2\right)^{-1/2} - \left[x^2+(d+y)^2\right]^{-1/2}\right]$




Use formula $\sigma = -\varepsilon_0 \left.\frac{d\Phi}{d\hat{n}}\right|_{surface}$, the normal derivative points outward, so based on our setup, it's in the direction of $y$.

$$\frac{dV}{dy} = kq\left[\left(-\tfrac{1}{\cancel{2}}\right)\left(x^2+(d-y)^2\right)^{-3/2}\cancel{2}(d-y)(-1) + \left[x^2+(d+y)^2\right]^{-3/2}\left(+\tfrac{1}{\cancel{2}}\right)\cancel{2}(d+y)\right]$$

$$= kq\left[\left(x^2+(d-y)^2\right)^{-3/2}(d-y) + \left(x^2+(d+y)^2\right)^{-3/2}(d+y)\right].$$

Evaluation at $y=0$ yields

$$\left.\frac{dV}{dy}\right|_{y=0} = kq\left[\left(x^2+d^2\right)^{-3/2}d + \left(x^2+d^2\right)^{-3/2}d\right]$$

$$= \boxed{2kqd\left(x^2+d^2\right)^{-3/2}}$$

$$\boxed{\sigma = -2\varepsilon_0 kqd\left(x^2+d^2\right)^{-3/2}}$$

Davidson Cheng

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